MICRO-428: Metrology

Week Ten: Electrical Metrology

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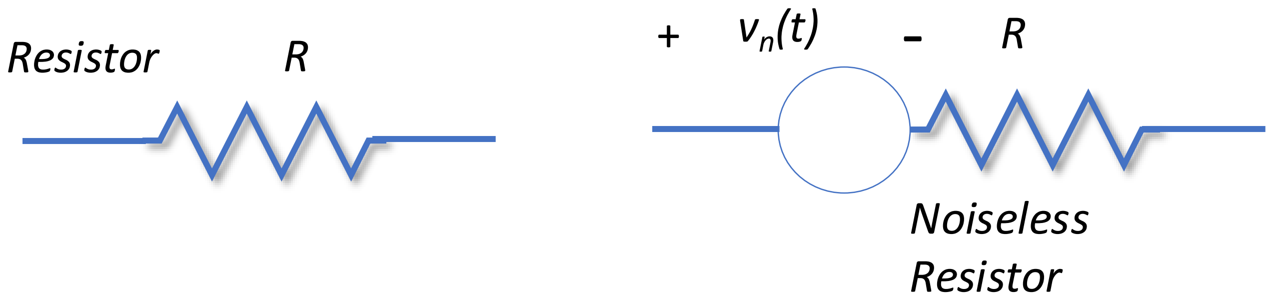
Exercise 1: Thermal Noise in Resistor

- Find the equivalent noise voltage of the resistor R

- Thermal noise on a resistor can be modeled by a series voltage source $v_n(t)$:



- Hint 1*: Noise power at temperature T delivered to an equal load resistor is assumed as: $P = kT\Delta f$
- Hint 2: Make use of the Maximum Power Transfer Theorem

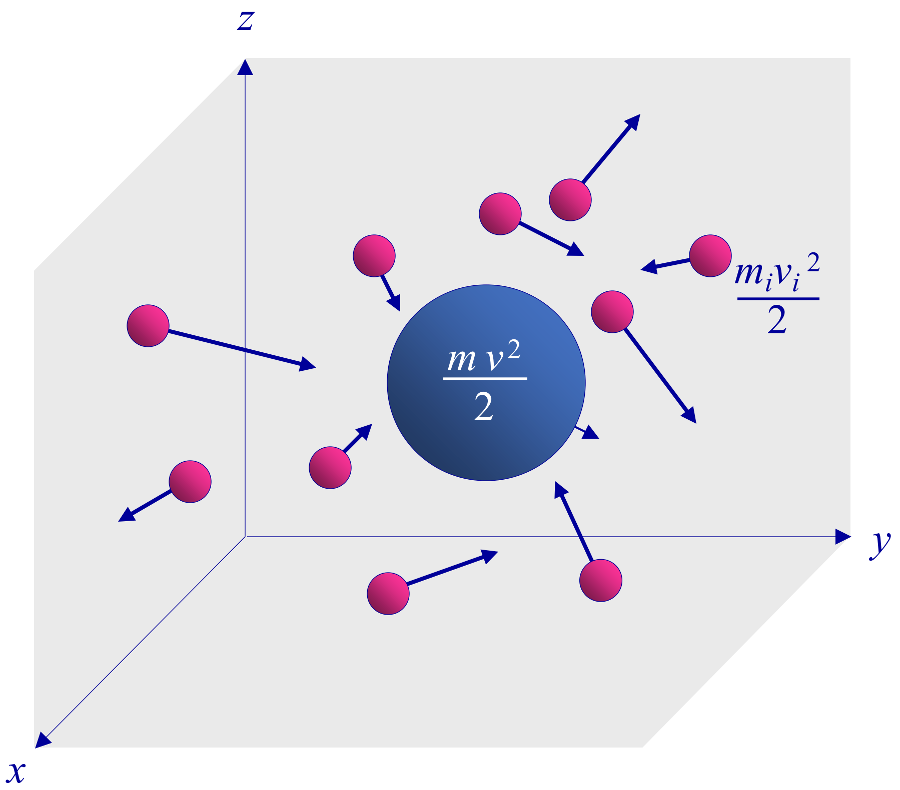
*C. Kittel, Elementary Statistical Physics, John Wiley, 1958.

Exercise 2: kTC Noise

Illustration: The law of the equipartition of energy

E. Paperno, BGU (IL), Measurement Theory Fundamentals, Ch. 5, 2006

Each particle has three degrees of freedom



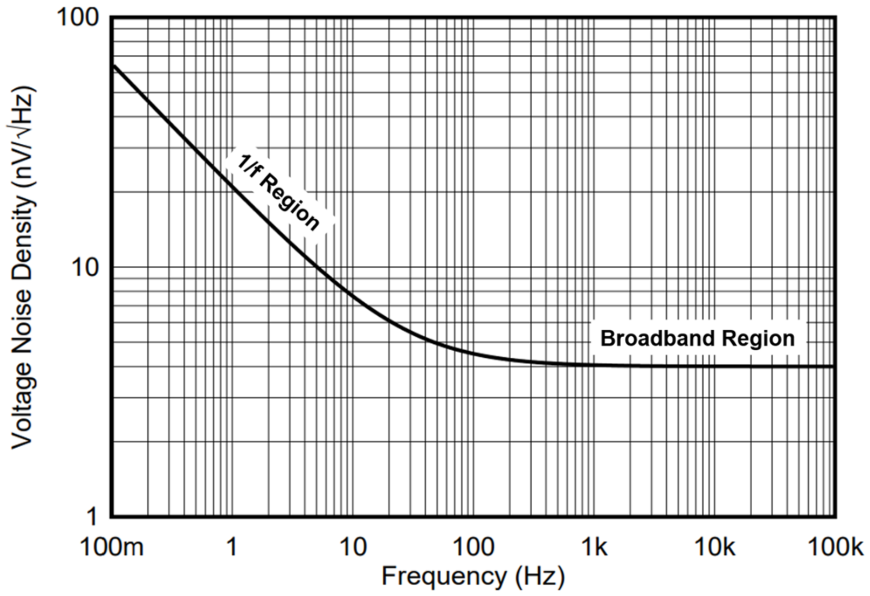
In thermal equilibrium:

$$\frac{m_i \overline{v_i^2}}{2} = \frac{m \overline{v^2}}{2} = 3\frac{kT}{2}$$

Exercise 3: 1/f noise: Scale invariance

- A 1/f noise is characterized by a power spectral density function:

$$S_F(f) = \frac{C}{f}$$

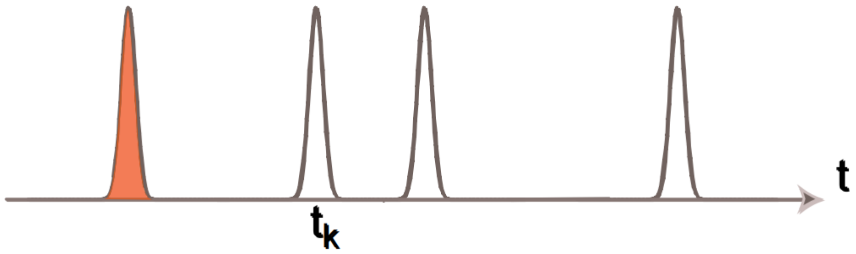


- Show that the total noise power in between any decade of frequency is constant.

P. Dutta and M. Horn, Rev. Mod. Phys. 53, 497 (1981).
M. B. Weissman, Rev. Mod. Phys. 60, 537 (1988).

Exercise 4: Shot noise derivation

- A series of random pulses are used to describe the current, with each pulse carrying a charge of e.



- Using the Wiener-Khinchin Theorem and assuming that electron passage follows a Poisson distribution, demonstrate that the spectral density of shot noise can be represented as (bilateral formulation):

$$S(f) = q|I|$$

Exercise 4: Shot noise (Wiener-Khinchin Relation)

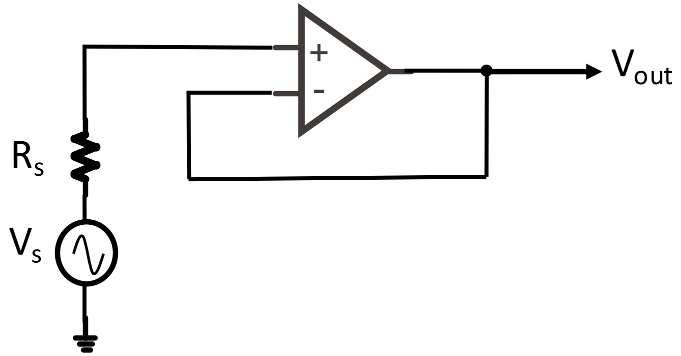
- *Reminder*: for a given wide sense stationary random process X(t), the auto-correlation function $K_{xx}(\tau)$ and its power density spectrum $S_{xx}(\omega)$ form a Fourier transform pair and are given by:

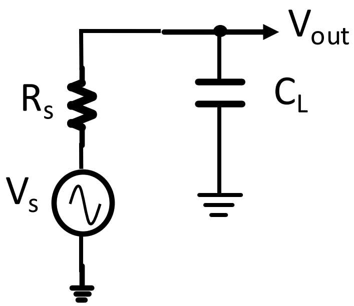
$$S_{xx}(\omega) = \int_{-\infty}^{+\infty} K_{xx}(\tau) e^{-j\omega\tau}\, d\tau$$

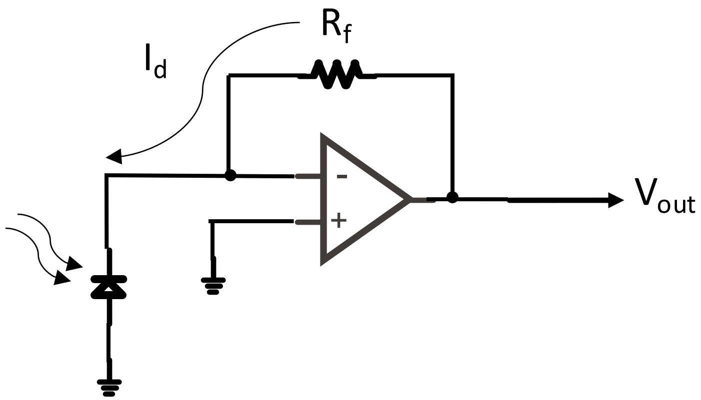
$$K_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{xx}(\omega) e^{j\omega\tau}\, d\omega$$

See Week10_Electrical Metrology, slide 20

Homework 1: Noise sources calculation







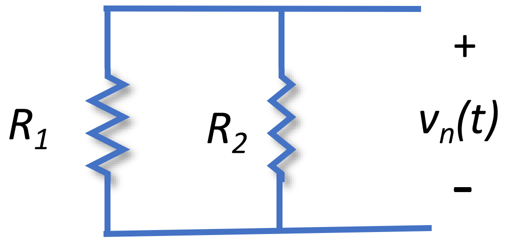
$R_s = 50\Omega, C_L = 1\,pF, I_d = 20\,\mu A, R_f = 5\,k\Omega$

Considering an infinite gain Opamp with a 100 MHz bandwidth, calculate the output rms noise voltage for the following cases:

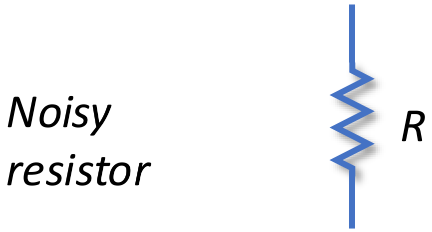
- Case 1: The amplifier doesn't generate any internal noise, and capacitive loading is ignored.
- Case 2: The buffer is removed and the source is loaded with C_L.
- Case 3: The amplifier doesn't generate any internal noise, and capacitive loading is ignored.

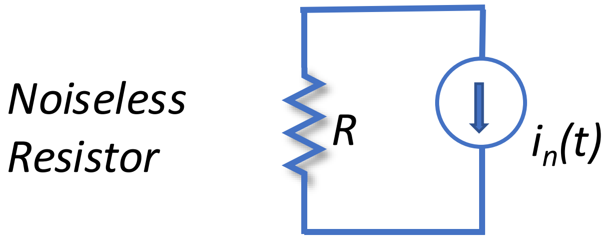
Homework 2: Thermal Noise in Resistor

- *Exercise*: Find the equivalent noise voltage of two resistors in parallel, R_1 and R_2



- *Hint:* Thermal noise of a resistor can be also represented by a parallel current source

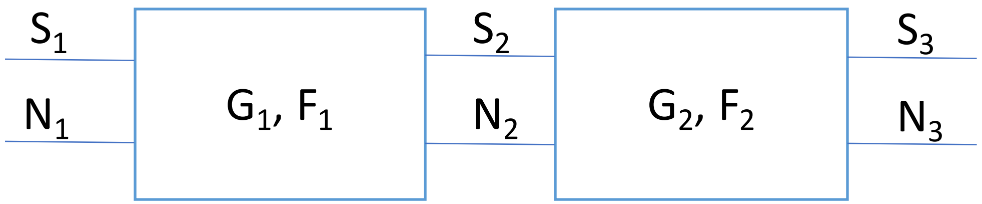




B. Razavi, Design of Analog CMOS Integrated Circuits, McGraw Hill., 2017

Homework 3: Noise factor of cascading two port network

- *Exercise*: a/ Find the equivalent noise factor F of the following network



N : Total noise power
S : Total signal power
G(f) : Gain in power
F(f) : Noise factor*

- *Hint:* * The noise factor of a two port network can be defined as :

$$F(f) = \frac{PSD\ of\ total\ noise\ at\ the\ output}{PSD\ at\ the\ output\ due\ to\ source\ resistance} = \frac{N_O(f)}{G(f) \cdot N_i(f)}$$

$$N_O = G \cdot N_i + (F-1) \cdot G \cdot N_i = F \cdot G \cdot N_i$$

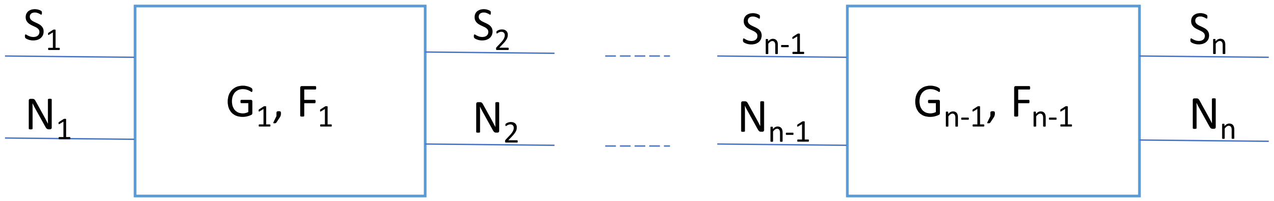
Amplified input noise — Internal noise

📖 C. Enz, HF and VHF CIRCUITS and TECHNIQUES 1

Homework 3: Noise factor of cascading two port network

- *Exercise*: b/ Generalize the noise factor formula for the following network (Friis formula).

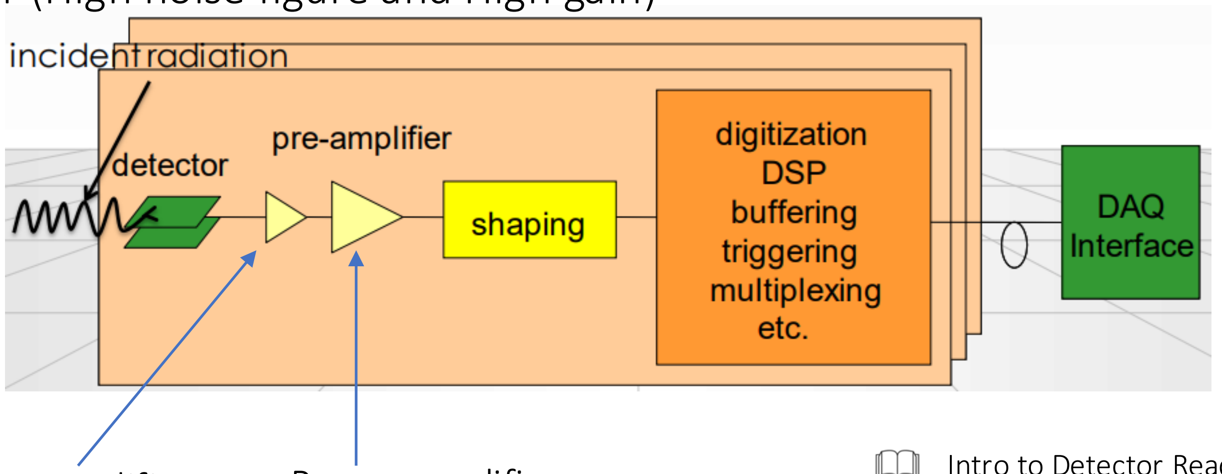
C. Enz, HF and VHF CIRCUITS and TECHNIQUES 1.



- c/ Give the right order of the amplification stages to minimize the total noise factor.

Stage A: Low noise amplifier (Low noise figure F, Moderate gain)

Stage B: Power amplifier (High noise figure and High gain)



Pre-amplifier

Power amplifier

Intro to Detector Readout ISOTDAQ 2012, N. Neufeld CERN/PH